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Characterizations of open and closed maps in a bigeneralized topological space (BGTS)*

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Abstract. The main thrust of this paper is to introduce and characterize closed and open functions in a bigeneralized topological space (BGTS). In particular, we establish the properties of $\mu^{(m,n)}$-$g^*\omega\alpha$ open (resp. $\mu^{(m,n)}$-$g^*\omega\alpha$ closed) functions in BGTS.

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1. Introduction

The concepts of generalized neighborhood systems and generalized topological spaces were first presented by Császár [7] in 2002. In addition, he introduced the ideas of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. Specifically, he characterized generalized continuous functions using a closure operator established on generalized neighborhood systems.

Later, in 2011, the idea of bigeneralized topological space (briefly BGTS) was then introduced by Boonpok [6] along with some of the fundamental properties of (m,n)-closed and (m,n)-open sets and weakly open functions.

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On the other hand, Levine [10] and Njastad [15] introduced some forms of open sets called *semi-open* and *α-open sets* in topological spaces. The notion of a *generalized closed (g-closed) se*t was defined by Levine [9] which led to the study of *generalized α-closed sets* and *α-generalized closed sets* by Maki et al. [12, 13] in 1970.

Sundaram and Sheik [17] also defined and studied *ω-closed sets* that open the idea of the recent study of Benchalli et al. [2, 3-5] on a weaker form of set, namely *$\omega\alpha$-closed set.*

In 2015, Baculta and Rara [2] introduced the fuzzy μ-regular generalized star b-sets in generalized fuzzy topological spaces. Further, Benchalli, Mirajakar, and Patil [5] introduced the idea of generalized star $\omega\alpha$-sets in a topological space. Their properties and characterizations were established.

Now, one of the interesting topics in the study of Topology is the concept of functions. Different types of functions have been investigated over time and up until the present, this notion continues to be a fascinating subject of inquiry in the field of Topology.

In this paper, we introduce another type of open and closed functions in BGTS using $g^*\omega\alpha$-sets defined by Benchalli et al. [5]. Moreover, we explore some properties and characterizations of these open and closed maps.

2. Preliminaries

In this section, the author recalls some preliminary concepts needed in the study. For standard terminologies and notations used in this study, the reader may refer to the Topology book by Dugundji [8].

The following definitions were first introduced by Császár [7] in 2008.

Definition 2.1 [7]. Let X be a nonempty set and denote the set of all subsets of X by $P(X)$. A subset μ of $P(X)$ is said to be a *generalized topology* (briefly GT) on X if $\varnothing \in \mu$ and the arbitrary union of elements of μ belongs to μ.

A GT μ is said to be a *strong generalized topology (briefly SGT)* if $X \in \mu$. If μ is a GT on X, then (X, μ) is said to be a *generalized topological space* (briefly GTS), and the elements of μ are called *μ-open sets*. The complement of μ-open sets are called *μ- closed sets*.

If $A \subseteq X$, then the *μ-closure* of A, denoted by $c_\mu(A)$, is the intersection of all μ-closed sets containing A. The *μ-interior* of A, denoted by $i_\mu(A)$, is the union of all μ-open sets contained in A.

Definition 2.2 [3,16]. A set A of a GTS (X, μ) is said to be:

(i) *μ-semi-open* if $A \subseteq c_\mu(i_\mu(A))$; the complement of *$\mu$- semi-open set* is *$\mu$-semi-closed*.

(ii) *μ-regular open* if $A{=}i_\mu(c_\mu(A))$; *μ-regular closed* if $A{=}c_\mu(i_\mu(A))$;

(iii) *μ-ω-closed set* if $c_\mu(A) \subseteq U$, whenever $A \subseteq U$ and U is *μ-semi-open* in X; the complement of a *μ-ω-closed set* is *μ-ω-open set*;

(iv) *μ-α-closed* if $c_\mu(i_\mu(c_\mu(A)) \subseteq A$;

(v) *μ-$\omega\alpha$-closed* if $\alpha c_\mu(A) \subseteq U$ whenever $A \subseteq U$ is *μ-ω-open* in X; the complement of a *μ-$\omega\alpha$-closed set* is *μ-$\omega\alpha$-open set*.

Definition 2.3 [11]. Let (X, μ) be a GTS and $A \subseteq X$. The *μ-α-closure* of A, denoted by $\alpha c_\mu(A)$, is the intersection of all μ-α-closed sets containing A. That is, $\alpha c_\mu(A) = \bigcap\{F \subseteq X : F \text{ is } \mu\text{-}\alpha\text{-closed and } A \subseteq F\}$. The *$\mu$-$\alpha$-interior* of A, denoted by $\alpha i_\mu(A)$, is the union of all μ-α-open sets contained in A. That is, $\alpha i_\mu(A) = \bigcup\{O \subseteq X : O \text{ is } \mu\text{-}\alpha\text{-open and } O \subseteq A\}$.

Definition 2.4 [14]. Let (X, μ) be a GTS and $A \subseteq X$. A is said to be μ-generalized star $\omega\alpha$-closed (briefly μ-$g^*\omega\alpha$-closed) set if $c_\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is μ-$\omega\alpha$-open in X. The complement of μ-$g^*\omega\alpha$-closed set is said to be μ-$g^*\omega\alpha$-open set. If A is both μ-$g^*\omega\alpha$-closed set and μ-$g^*\omega\alpha$-open set, then A is said to be μ-$g^*\omega\alpha$-clopen set.

Definition 2.5 [14]. Let (X, μ) be a GTS and $A \subseteq X$. The union of all the μ-$g^*\omega\alpha$-open sets contained in A is called the μ-$g^*\omega\alpha$-interior of A denoted by $g^*\omega\alpha i_\mu(A)$.The intersection of all the μ-$g^*\omega\alpha$-closed sets containing A is called the μ-$g^*\omega\alpha$-closure of A denoted by $g^*\omega\alpha c_\mu(A)$.

Theorem 2.6 [14]. *Every μ-closed set is μ-$g^*\omega\alpha$-closed.*

The next corollary is immediate from Theorem 2.6.

Corollary 2.7 [14]. *Every μ-open set is μ-$g^*\omega\alpha$-open.*

Throughout this paper, m and n are elements of the set $\{1, 2\}$ where $m \neq n$.

Definition 2.8 [6]. Let X be a nonempty set and let μ_1, μ_2 be generalized topologies on X. The triple (X, μ_1, μ_2) is said to be a *bigeneralized topological space* (briefly BGTS).

Definition 2.9 [1]. Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be a function. Then f is said to be $\mu^{(m,n)}$-*open* (resp. $\mu^{(m,n)}$-*closed*) if for each μ_X^m-open set (resp. μ_X^m-closed) V in X, $f(V)$ is μ_Y^n-open (resp. μ_Y^n-closed) in Y.

3. Results

In this section, we establish some properties and characterization of $\mu^{(m,n)}$-$g^*\omega\alpha$ open (resp. $\mu^{(m,n)}$-$g^*\omega\alpha$ closed) functions in BGTS. All throughout this section, (X, μ_X^1, μ_X^2) and (Y, μ_Y^1, μ_Y^2) are BGTS.

This section aims to introduce and discuss the concepts of $\mu^{(m,n)}$-$g^*\omega\alpha$

open and $\mu^{(m,n)}$-$g^*\omega\alpha$ closed functions.

Definition 3.1. Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be a function. Then f is said to be:

(i) $\mu^{(m,n)}$-$g^*\omega\alpha$ *open* (resp. $\mu^{(m,n)}$-$g^*\omega\alpha$ *closed*) if for each μ_X^m-open set (resp. μ_X^m-closed) V in X, $f(V)$ is μ_Y^n-$g^*\omega\alpha$ open (resp. μ_Y^n-$g^*\omega\alpha$ closed) in Y.

(ii) *pairwise* μ-$g^*\omega\alpha$ *open* (resp. *pairwise* μ-$g^*\omega\alpha$ *closed*) if f is both quasi $\mu^{(1,2)}$-$g^*\omega\alpha$ open (resp. $\mu^{(1,2)}$-$g^*\omega\alpha$ closed) and $\mu^{(2,1)}$-$g^*\omega\alpha$ open (resp. $\mu^{(2,1)}$-$g^*\omega\alpha$ closed).

Example 3.2. Let $X = \{u, v, w\}$ and $Y = \{a, b, c\}$. Consider the GT's μ_X^1=$\{\varnothing,\ \{u, v\},\ X\}$, μ_X^2=$\{\varnothing,\ \{u\},\ \{u, v\}\}$, μ_Y^1=$\{\varnothing,\ \{a\},\ \{a, b\},\ Y\}$, and μ_Y^2=$\{\varnothing,\ \{a, b\},\ \{b, c\},\ Y\}$.

Observe that the μ_X^1-closed sets in X are X, $\{w\}$ and $\varnothing$ and the μ_X^2-closed sets are X, $\{v, w\}$, and $\{w\}$. Also, the μ_Y^1-$g^*\omega\alpha$ open sets in Y are $\varnothing$, Y, $\{a, b\}$, and $\{a\}$ and the μ_Y^2-$g^*\omega\alpha$ open sets are $\varnothing$, Y, $\{b, c\}$, $\{a, b\}$ and $\{a\}$.

Further, the μ_Y^2-$g^*\omega\alpha$ closed sets are $\varnothing$, Y, $\{c\}$, and $\{b, c\}$ and the μ_Y^2-$g^*\omega\alpha$ closed sets are $\varnothing$, Y, $\{a\}$, $\{c\}$ and $\{b, c\}$.

Let $f : (Y, \mu_X^1, \mu_X^2) \to (X, \mu_Y^1, \mu_Y^2)$ be defined by $f(u)$=a, $f(v)$=b, and $f(w)$=c. Then f is $\mu^{(1,2)}$-$g^*\omega\alpha$ open and f is $\mu^{(2,1)}$-$g^*\omega\alpha$ open.

It follows that f is pairwise μ-$g^*\omega\alpha$ open. Further, since f is $\mu^{(1,2)}$-$g^*\omega\alpha$ closed and $\mu^{(2,1)}$-$g^*\omega\alpha$ closed, f is pairwise μ-$g^*\omega\alpha$ closed.

Theorem 3.3. *If a function $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is $\mu^{(m,n)}$-open (resp. $\mu^{(m,n)}$-closed), then f is $\mu^{(m,n)}$-$g^*\omega\alpha$ open (resp. $\mu^{(m,n)}$-$g^*\omega\alpha$ closed).*

Proof. Let V be μ_X^m-open (resp. μ_X^m-closed) set in X. Since f is $\mu^{(m,n)}$-open (resp. $\mu^{(m,n)}$-closed), $f(V)$ is μ_Y^n-open (resp. μ_Y^n-closed) in Y.

By Corollary 2.7 and Theorem 2.6, $f(V)$ is μ_Y^n-$g^*\omega\alpha$ open (resp. μ_Y^n-$g^*\omega\alpha$ closed) in Y. Therefore, f is $\mu^{(m,n)}$-$g^*\omega\alpha$ open (resp.$\mu^{(m,n)}$-$g^*\omega\alpha$ closed). □

The next theorem characterizes a $\mu^{(m,n)}$-$g^*\omega\alpha$ open function.

Theorem 3.4. *A function $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is $\mu^{(m,n)}$-$g^*\omega\alpha$ open if and only if for every subset A of Y and for every μ_X^m-closed set F in X containing $f^{-1}(A)$, there exists a μ_Y^n-$g^*\omega\alpha$ closed set B in Y containing A such that $f^{-1}(B) \subseteq F$.*

Proof. Suppose that f is $\mu^{(m,n)}$-$g^*\omega\alpha$ open. Let $A \subseteq Y$ and let F be μ_X^m-closed set of X such that $f^{-1}(A) \subseteq F$. Since F^c is μ_X^m-open in X, $f(F^c)$ is μ_Y^n-$g^*\omega\alpha$ open in Y.

Let B=$[f(F^c)]^c$.

Hence, B is μ_Y^n-$g^*\omega\alpha$ closed in Y. Since $f^{-1}(A) \subseteq F$, $F^c \subseteq [f^{-1}(A)]^c$= $f^{-1}(A^c)$. Thus, $f(F^c) \subseteq A^c$.

Consequently, $[(A^c)]^c \subseteq [f(F^c)]^c$=$B$ which implies that $A \subseteq B$. Hence,

$$f^{-1}(B) = f^{-1}([f(F^c)]^c) = [f^{-1}(f(F^c))]^c \subseteq [F^c]^c = F.$$

Therefore, $f^{-1}(B) \subseteq F$.

Conversely, let O be μ_X^m-open in X. Now, $f^{-1}[(f(O))]^c$=$[f^{-1}(f(O))]^c \subseteq O^c$. Since O^c is μ_X^m-closed set in X, by assumption, there exists a μ_Y^n-$g^*\omega\alpha$ closed set B of Y such that $[f(O)]^c \subseteq B$ and $f^{-1}(B) \subseteq O^c$ so that $O \subseteq [f^{-1}(B)]^c$.

Thus, $B^c \subseteq f(O) \subseteq f([f^{-1}(B)]^c)$=$f(f^{-1}(B^c)) \subseteq B^c$.

Hence, $f(O)$=B^c. Since B^c is μ_Y^n-$g^*\omega\alpha$ open in Y, $f(O)$ is μ_Y^n-$g^*\omega\alpha$

open in Y. Therefore, f is $\mu^{(m,n)}$-$g^*\omega\alpha$ open. □

Theorem 3.5. *If $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is a $\mu^{(m,n)}$-$g^*\omega\alpha$ open function, then $f(i_{\mu_X^m}(A)) \subseteq g^*\omega\alpha i_{\mu_Y^n}(f(A))$ for every $A \subseteq X$.*

Proof. Let $A \subseteq Y$. Since $i_{\mu_X^m}(A)$ is μ_X^m-open in X and f is $\mu^{(m,n)}$-$g^*\omega\alpha$ open, $f(i_{\mu_X^m}(A))$ is μ_Y^n-$g^*\omega\alpha$ open in Y.

Moreover, $i_{\mu_X^m}(A) \subseteq A$ implies that $f(i_{\mu_X^m}(A)) \subseteq f(A)$. However, $g^*\omega\alpha i_{\mu_Y^n}(f(A))$ is the union of all μ_Y^n-$g^*\omega\alpha$ open set in Y contained in $f(A)$. Hence, $f(i_{\mu_X^m}(A)) \subseteq g^*\omega\alpha i_{\mu_Y^n}(f(A))$. □

Theorem 3.6. *If $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is $\mu^{(m,n)}$-$g^*\omega\alpha$ open, then $f^{-1}(g^*\omega\alpha c_{\mu_Y^n}(B)) \subseteq c_{\mu_X^m}(f^{-1}(B))$ for every subset B in Y.*

Proof. Let B be any subset of Y. Since $f^{-1}(B) \subseteq c_{\mu_X^m}(f^{-1}(B))$ and $c_{\mu_X^m}(f^{-1}(B))$ is μ_X^m-closed in X, by Theorem 3.4, there exists a μ_Y^n - $g^*\omega\alpha$ closed set F of Y such that $B \subseteq F$ and $f^{-1}(F) \subseteq c_{\mu_X^m}(f^{-1}(B))$.

Since $B \subseteq F$ and F is μ_Y^n-$g^*\omega\alpha$ closed, $g^*\omega\alpha c_{\mu_Y^n}(B) \subseteq g^*\omega\alpha c_{\mu_Y^n}(F) = F$.

Thus, $f^{-1}(g^*\omega\alpha c_{\mu_Y^n}(B)) \subseteq f^{-1}(F)$. Since $f^{-1}(F) \subseteq c_{\mu_X^m}(f^{-1}(B))$, we have $f^{-1}(g^*\omega\alpha c_{\mu_Y^n}(B)) \subseteq c_{\mu_X^m}(f^{-1}(B))$. □

The following theorem characterizes the $\mu^{(m,n)}$-$g^*\omega\alpha$ open functions in BGTS.

Theorem 3.7. *Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be a bijective function. Then f is $\mu^{(m,n)}$-$g^*\omega\alpha$ open if and only if f is $\mu^{(m,n)}$-$g^*\omega\alpha$ closed.*

Proof. Let V be a μ_X^m-open set in X. Then V^c is μ_X^m-open in X. Since f is $\mu^{(m,n)}$-$g^*\omega\alpha$ open, $f(V^c)$ is μ_Y^n-$g^*\omega\alpha$ open in Y. Since f is bijective, $[f(V)]^c$=$f(V^c)$ is μ_Y^n-$g^*\omega\alpha$ open in Y.

It follows that $f(V)$ is μ_Y^n-$g^*\omega\alpha$ closed in Y. Consequently, f is $\mu^{(m,n)}$-$g^*\omega\alpha$ closed.

Conversely, suppose that f is $\mu^{(m,n)}$-$g^*\omega\alpha$ closed.

Let U be μ_X^m-open in X. Then U^c is μ_X^m closed in X and $f(U^c)$ is μ_Y^n-$g^*\omega\alpha$ closed in Y.

Since f is bijective, $[f(U)]^c$=$f(U^c)$ is μ_Y^n-$g^*\omega\alpha$ closed in Y. Thus, $f(U)$ is μ_Y^n-$g^*\omega\alpha$ open in Y. Therefore, f is $\mu^{(m,n)}$-$g^*\omega\alpha$ open. □

The next result characterizes the $\mu^{(m,n)}$-$g^*\omega\alpha$ closed functions in BGTS.

Theorem 3.8. *A function $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is $\mu^{(m,n)}$-$g^*\omega\alpha$ closed if and only if for every subset S of Y and for every μ_X^n-open set O of X containing $f^{-1}(S)$, there exists a μ_Y^m-$g^*\omega\alpha$ open set U of Y containing S such that $f^{-1}(U) \subseteq O$.*

Proof. Suppose that f is a $\mu^{(m,n)}$-$g^*\omega\alpha$ closed function. Let $S \subseteq Y$ and O be a μ_X^m-open set of X such that $f^{-1}(S) \subseteq O$. Then O^c is μ_X^m-closed in X and $f(O^c)$ is μ_Y^n-$g^*\omega\alpha$ closed set in Y. Let U=$[f(O^c)]^c$.

Hence, U is μ_Y^n-$g^*\omega\alpha$ open. Since

$$f^{-1}(S) \subseteq O,$$
$$O^c \subseteq [f^{-1}(S)]^c = f^{-1}(S^c).$$

Thus, $f(O^c) \subseteq f(f^{-1}(S^c)) \subseteq S^c$.

It follows that $S \subseteq U$ and

$$f^{-1}(U) = f^{-1}[f(O^c)]^c = [f^{-1}(f(O^c))]^c \subseteq [O^c]^c = O.$$

Therefore, $f^{-1}(U) \subseteq O$.

Conversely, let V be μ_X^m-closed set in X. Now,

$$f^{-1}([f(V)]^c) = [f^{-1}(f(V))]^c \subseteq V^c.$$

Since V^c is μ_X^m-open in X, by assumption, there exists a μ_Y^n-$g^*\omega\alpha$ open set U of Y such that $[f(V)]^C \subseteq U$ and $f^{-1}(U) \subseteq V^c$.

Hence, $V \subseteq [f^{-1}(U)]^c$. Thus,

$$U^c \subseteq f(V) \subseteq f([f^{-1}(U)]^c) = f(f^{-1}(U^c)) \subseteq U^c.$$

Then, $f(V) = U^c$. Since U is a μ_Y^n-$g^*\omega\alpha$ open set in Y, $f(V) = U^c$ is μ_Y^n-$g^*\omega\alpha$ closed in Y.

Therefore, f is a $\mu^{(m,n)}$-$g^*\omega\alpha$ closed function. □

Theorem 3.9. *If a function $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is $\mu^{(m,n)}$-$g^*\omega\alpha$ closed, then $g^*\omega\alpha c_{\mu_Y^n}(f(A)) \subseteq f(c_{\mu_X^m}(A))$ for every subset A of Y.*

Proof. Suppose that f is $\mu^{(m,n)}$-$g^*\omega\alpha$ closed. Let $A \subseteq Y$. Since $c_{\mu_X^m}(A)$ is μ_X^m-closed in X, $f(c_{\mu_X^m}(A))$ is μ_Y^n-$g^*\omega\alpha$ closed in Y.

Moreover, since $A \subseteq c_{\mu_X^m}(A)$, $f(A) \subseteq f(c_{\mu_X^m}(A))$. However, $g^*\omega\alpha c_{\mu_Y^n}(f(A))$ is the intersection of all μ_Y^n-$g^*\omega\alpha$ closed set in Y containing $f(A)$. Thus, $g^*\omega\alpha c_{\mu_Y^n}(f(A)) \subseteq f(c_{\mu_X^m}(A))$. □

4. Conclusion

In this paper, we have introduced stronger notions of open and closed maps in a bigeneralized topological space, stronger versions of the work done by Baculta and Rara [1]. In particular, we have shown that if a function $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is $\mu^{(m,n)}$-*open* (resp. $\mu^{(m,n)}$-*closed*), then f is $\mu^{(m,n)}$-$g^*\omega\alpha$-*open* (resp. $\mu^{(m,n)}$-$g^*\omega\alpha$-*closed*).

Moreover, f is $\mu^{(m,n)}$-$g^*\omega\alpha$-*open* if and only if for every subset A of Y and for every μ_X^m-*closed* set F in X containing $f^{-1}(A)$, there exists a μ_Y^n-$g^*\omega\alpha$-*closed* set B in Y containing A such that $f^{-1}(B) \subseteq F$. In addition, if f admits a bijection, then f is $\mu^{(m,n)}$-$g^*\omega\alpha$-*open* if and only if f is $\mu^{(m,n)}$-$g^*\omega\alpha$-*closed.*

Lastly, we have shown that f is $\mu^{(m,n)}$-$g^*\omega\alpha$-*closed* if and only if for every subset S of Y and for every μ_X^n-*open* set O of X containing $f^{-1}(S)$, there exists a μ_Y^m-$g^*\omega\alpha$-*open* set U of Y containing S such that $f^{-1}(U) \subseteq O$.

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